NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS. MAY 28TH 2019. TITLES AND ABSTRACTS OF THE TALKS

Thierry Cazenave: Finite-time blowup for a Schrödinger equation with a nonlinear source term.

Abstract: In this joint work with Zheng Han, Yvan Martel and Lifeng Zhao, we consider the nonlinear Schrödinger equation \( u_t = i\Delta u + |u|^\alpha u \) on \( \mathbb{R}^N \), for \( H^1 \)-subcritical or critical nonlinearities: \( \alpha > 0 \) and \( (N - 2)\alpha \leq 4 \). This equation combines two important properties: the associated ODE \( u' = |u|^\alpha u \) produces finite-time blowup; and the equation can be solved backwards in time. Using these properties we prove that, given any compact set \( E \subset \mathbb{R}^N \), there exist finite-energy solutions which are defined on some time interval \( (-T,0) \) and blow up at \( t = 0 \) exactly on \( E \). The construction is based on an appropriate ansatz. The initial ansatz (which is sufficient when \( \alpha > 1 \)) is simply \( U_0(t,x) = \kappa (t + A(x))^{-\frac{1}{\alpha}} \), where \( A \geq 0 \) vanishes exactly on \( E \), which is a solution of the ODE \( u' = |u|^\alpha u \). If \( \alpha \leq 1 \), we need to refine this ansatz, and we proceed inductively, using only ODE techniques. We complete the proof by energy estimates and a compactness argument. We prove similar results for the nonlinear wave equation, which has a comparable structure (finite-time blowup for the associated ODE, and time-reversibility).

Mahir Hadzic: On gravitational collapse of Newtonian stars.

Abstract: We prove the existence of an infinite-dimensional family of radial collapsing star solutions of the gravitational Euler-Poisson system, with a prescribed space-time collapse curve. If time permits we also discuss the question of stable collapse. This is a joint work with Y. Guo and J. Jang.

Jérémie Szeftel: The nonlinear stability of Schwarzschild.

Abstract: I will discuss a joint work with Sergiu Klainerman on the stability of Schwarzschild as a solution to the Einstein vacuum equations with initial data subject to a certain symmetry class.